



## Uncertainty Analysis Applied to Dredge Production Calculation

**INTRODUCTION:** The accurate measurement of dredge production is essential for maintaining the maximum efficiency and cost effectiveness of the dredging process. The use of production measurement systems on pipeline and hopper dredges provides dredging personnel with tools for measuring and monitoring production quantities. The accuracy of these production monitoring systems varies according to the instrumentation used and the knowledge of the sediment and water properties associated with the dredging activity.

Each measurement and physical quantity associated with the calculation of dredging quantities has some error or uncertainty associated with it. The equations for calculating production are functions of multiple variables (measurements and physical quantities), each contributing some error. These errors propagate through the data reduction equation to the final calculation. It is essential that the error associated with each variable is accounted for, and that the individual error contribution to the total error is recognized.

Equations are introduced that describe production for both pipeline and hopper dredges. An uncertainty analysis expression is derived for each equation (Scott 1993). The general uncertainty analysis technique is applied in a step-by-step manner to one of the equations to show the derivation of the uncertainty analysis expression. Example dredging situations are introduced to demonstrate the uncertainty analysis application. Numerical solutions are obtained which show the error contribution of each variable, and the effect of uncalibrated instruments and unmeasured sediment and water properties on the accuracy of production calculations. Uncertainty analysis calculations indicate that with properly calibrated instruments and measured sediment and water properties, the error associated with production measurements can be 10 percent or less. If the instruments are not calibrated and the sediment and water properties are not measured, an error potential of 25 to 50 percent is possible in production calculations.

**BACKGROUND:** The overall efficiency of dredging operations is directly related to the production rate of dredged materials. The dredging process is optimized when there is maximum production at the lowest operating cost. It is essential that dredging personnel are informed of the optimum operating conditions of the dredge plant, as well as the capability of the production monitoring system. The use of production monitoring instrumentation has provided dredging personnel with a useful tool for monitoring dredge production and overall dredge operation. Production monitoring equipment is primarily utilized on pipeline dredges which dispose of the dredged materials through a pipeline, or by hopper dredges, which store the dredged material in onboard hoppers for later disposal.

Two types of production monitoring systems are commonly used on either pipeline or hopper dredges. An in-line production meter system directly measures the density and flow velocity of the material in the dredge pipe. This system utilizes a nuclear density gauge for determining the density

of the slurry, and either a doppler or magnetic flow meter for measuring the flow velocity in the pipe. Typically, the signals from these instruments are processed through a cross-point display unit which consists of two pointers that indicate the optimum production based on the pointer positions (slurry density and flow velocity).

For hopper dredges, the production is determined by measuring the average density of the load in the hopper. This is accomplished by relating the draft of the hopper barge to the weight of material in the hopper. The draft measurements are generally made by differential pressure transducers located in the bottom of the hull of the vessel. The hopper volume at any time is determined by water-level sensors mounted above the hopper. By knowing the weight of the dredged material in the hopper and the hopper volume, the average density of the material in the hopper can be calculated.

Real-time production data such as flow rate of dredged solids (kg/hr), and flow rate of dredged in situ volumes (m<sup>3</sup>/hr) for in-line production meter systems, and total weight (kg) and total in situ volume (m<sup>3</sup>) for hopper monitoring systems can be made available to dredging personnel by processing the signals from the instruments through a data acquisition system linked to a personal computer. These signals are input into computer codes which calculate the production of dredged material.

**PURPOSE:** The data reduction equations used to calculate the production quantities associated with pipeline and hopper dredges contain variables that introduce error into the final production calculation. These variables include not only the measurements made by the instrumentation, but also those associated with the dredging environment such as the density of the water and dredged sediments. The error due to one variable may be insignificant, but the propagation of the error through a data reduction equation with multiple variables may result in excessive uncertainty or error in the final result. This technical note will present an analytical method for determining the influence of each variable in the production equation on the total error resulting from the production calculation. Equations defining both solids and in situ volumetric production will be introduced. A mathematical method for determining the percent error or uncertainty of production calculations using these equations will be discussed and applied to example dredging problems.

**PRODUCTION CALCULATIONS:** The in-line production meter system provides production data in the form of solids flow rate or volumetric flow rate, with solids flow rate referring to only the flow rate of solids (weight per unit time) in the pipe, and volumetric flow rate referring to the volume of in situ material flowing through the pipe (volume per unit time). The equation for the in situ volumetric flow rate is defined by:

$$VOL(t) = \frac{\rho_s - \rho_w}{\rho_i - \rho_w} (V)(A) \quad (1)$$

where

$\rho_i$  = the in situ dredged sediment density

$\rho_s$  = the slurry density measured by the nuclear density probe

- $\rho_w$  = the density of the water  
 $V$  = the flow velocity in the pipe  
 $A$  = the area of the dredge pipe

The solids flow rate in the dredge pipe is defined as:

$$M(t) = \frac{\rho_s - \rho_w}{\rho_m - \rho_w} \rho_m (V)(A) \quad (2)$$

with  $\rho_m$  the sediment mineral density. The load in a dredge hopper can be defined in terms of volume of in situ materials or weight of sediments in the hopper. The volume of in situ material is computed by the following expression:

$$VOL = \frac{\rho_h - \rho_w}{\rho_i - \rho_w} (VOL_h) \quad (3)$$

with the solids load in a dredge hopper computed by:

$$M = \frac{\rho_h - \rho_w}{\rho_m - \rho_w} (\rho_m) (VOL_h) \quad (4)$$

with  $VOL_h$  the hopper volume and  $\rho_h$  the average density of the material in the hopper, calculated by dividing the weight of the material in the hopper as measured by the draft sensors located in the hull of the vessel, by the volume of material in the hopper as measured by the water level sensors mounted above the hopper.

The data reduction equations for the in-line production meter contain a total of six variables; the density of the water ( $\rho_w$ ), the sediment mineral density ( $\rho_m$ ), the in situ density of the sediments ( $\rho_i$ ), the velocity as measured by the flow meter ( $V$ ), the slurry density measured by the nuclear density gauge ( $\rho_s$ ), and the pipe diameter ( $D$ ).

The data reduction equations for the hopper production monitoring system contain a total of five variables; the density of the water ( $\rho_w$ ), the sediment mineral density ( $\rho_m$ ), the in situ density of the sediments ( $\rho_i$ ), and the average density of the material in the hopper,  $\rho_h$ . As mentioned before, this average density measurement is calculated by the weight of the material in the hopper ( $W_h$ ) divided by the volume of the material in the hopper ( $VOL_h$ ).

Each of the previously mentioned variables have some error associated with their values. This error may be associated with changing physical conditions in the dredging environment such as water temperature and salinity levels and variations in the mineral and organic content of the sediments, or measurement error inherent in the instrumentation. The error contributed by each variable will propagate through the production equations into the final production calculation.

**VARIABLE UNCERTAINTIES:** Each variable in the production calculation contains some measurement uncertainty. The following provides a brief explanation of the potential uncertainty associated with these variables.

**Water Density.** The water found within the dredging environment can vary in density due to dissolved and suspended solids content and temperature changes. The density of the water can generally vary within the range of 0.98 to 1.030 g/cm<sup>3</sup> because of the previously described conditions. The maximum error introduced into the production calculations from changes in water density, without compensation, is about 3 percent (Rokosch 1989).

**Sediment Mineral Density.** The types of sediment minerals found at dredging sites will vary according to the physical environment. Generally, coarse grained sediments such as sands and gravel will exist in riverine or coastal environments, while the finer grained materials such as silts and clays will be found in areas such as ports and bays, which have a more suitable environment for the settling of finer grained sediments. The density values for sands and gravel will generally vary within 2.65 to 2.67 g/cm<sup>3</sup>. Cohesive soils such as silts and clays can vary in density between 2.68 to about 2.75 g/cm<sup>3</sup>. Assuming a mineral density of 2.65 g/cm<sup>3</sup> in production calculations can generally result in an error range of 0-4 percent.

**In Situ Sediment Density.** Accurate measurement of the in situ sediment density is essential for the accurate calculation of volumetric production. The density of saturated sediments is dependent on the mineral density and the pore volume that the water occupies. A wide variety of in situ conditions exist which can have a significant influence on the density of the sediments. Uniform sands existing in a loose or dense state can have densities within the range of 1.89 to 2.09 g/cm<sup>3</sup> (Peck and Hanson 1967). Mixed sands (fine, medium, and coarse) in a loose or dense state can have densities within the range of 1.99 to 2.16 g/cm<sup>3</sup>. For finer sediments such as soft silts and clays with organic content, the density can range from 1.4 to 1.58 g/cm<sup>3</sup>. Fluid mud layers can be established at densities as low as 1.2 g/cm<sup>3</sup>, while fine, consolidated sediments such as stiff clays can have a density as high as 2.07 g/cm<sup>3</sup>. Dredging in mixed sediments with layers of fine grained sediments and coarse sediments can produce significant error if in situ density measurements are not taken and incorporated into the production calculations.

**Flow Meter Velocity Measurements.** The velocity of the sediment slurry flowing in the dredge pipe is generally measured by either a Doppler or magnetic flow meter. The Doppler flow meter is generally considered the least precise of the two. It is a noninvasive ultrasonic flow meter which attaches to the outside of the pipe. The accuracy of the Doppler meter as claimed by the manufacturer is  $\pm 2$  percent of full scale. The magnetic flow meter attaches to the dredge pipe, with an electrode penetrating the pipe lining. The manufacturer of these meters claims an accuracy of  $\pm 0.25$  percent of full-scale.

**Nuclear Density Measurements.** Nuclear density gauges are devices which measure radiation particle attenuation through a material. For dredging applications, the gauge attaches to the discharge pipe. A radioactive source emits particles through the pipe, and a receiver on the other side of the pipe counts the particles that pass through. The density of the slurry is measured by the attenuation of the particles as they pass through the slurry in the pipe. These are very accurate devices for measuring density of a homogeneous fluid, with a calibrated accuracy of  $\pm 0.001$  g/cm<sup>3</sup>. Fine

sediments (silts and clays) transported in large pipelines in fully turbulent flow are considered non-settling (homogeneously distributed). Sand-sized sediments stratify in the pipeline depending on particle size and concentration. A properly mounted density gauge is oriented at a 45-deg angle from the horizontal to insure that a representative sediment concentration profile is sampled. The larger the median grain size transported, the higher the degree of uncertainty in the density gauge measurement. For coarse sediment transport, the uncertainty could be much higher.

**Differential Pressure Transducers.** The pressure transducers located on the bottom of the hull of the vessel and used to measure the draft of the vessel because of the load in the hopper have accuracies of about  $\pm 1$  percent of the range of measurement when used for dredging applications. The actual calibrated accuracy of these transducers may be better than 1 percent, but additional error is introduced because the draft of the vessel is influenced by other factors such as the amount of fuel and ballast that the vessel is carrying, as well as the motion of the vessel caused by wave action.

**Water Surface Elevation Transducers.** The transducers designed to measure the surface of the material in large dredge hoppers operate on either ultrasonic or microwave signal transmission and reception principles. The accuracy of these transducers is estimated to be about 1 percent of the range of measurement for dredging applications.

**Measurement of the Diameter of the Dredge Pipe.** There is some assumed error in the measurement of the diameter of the discharge pipe on a dredge. Because there are many makes of pipe used in dredging, an assumed error of  $\pm 0.00025$  m will be used for the following error analysis.

**GENERAL UNCERTAINTY ANALYSIS:** A general uncertainty analysis is a mathematical method of determining how the error associated with each variable in a data reduction equation (such as a production equation) propagate through the equation to the final calculated result. For the case of the production calculation equations, the variables and their associated error are the water density, sediment mineral density, in situ sediment density, slurry density in the pipeline, flow velocity measurements, pipe diameter measurements, and average density in the hopper (pressure transducer and water level transducer measurements).

A detailed description of the principles and theory of the general uncertainty analysis technique is given by Coleman and Steel (1989). For the purpose of this document, only the basic uncertainty analysis expression will be described along with a procedural method for solving the expression for the desired result.

To demonstrate the general uncertainty analysis technique, the equation describing the in situ volumetric flow rate production will be analyzed step-by-step using this method. Earlier in the document, the equation for the volumetric flow rate of in situ materials measured by an in-line production meter was defined as:

$$VOL(t) = \frac{\rho_s - \rho_w}{\rho_i - \rho_w} (V)(A) \quad (5)$$

This equation is a function of five variables: the measured slurry density in the pipe ( $\rho_s$ ), the in situ density of the sediment ( $\rho_i$ ), the density of the water ( $\rho_w$ ), the flow velocity in the pipe ( $V$ ), and the pipe diameter ( $D$ ). This equation can be represented in the form:

$$VOL(t) = f(X_1, X_2, X_3, X_4, X_5) \quad (6)$$

which states that the in situ volumetric flow rate in the dredge pipe is a function of five independent variables:  $X_1$ ,  $X_2$ ,  $X_3$ ,  $X_4$ , and  $X_5$ . The uncertainty in the production calculation is given by the expression:

$$U_{VOL(t)} = \left[ \left( \frac{\partial VOL(t)}{\partial X_1} U_{X_1} \right)^2 + \left( \frac{\partial VOL(t)}{\partial X_2} U_{X_2} \right)^2 + \left( \frac{\partial VOL(t)}{\partial X_3} U_{X_3} \right)^2 + \left( \frac{\partial VOL(t)}{\partial X_4} U_{X_4} \right)^2 + \left( \frac{\partial VOL(t)}{\partial X_5} U_{X_5} \right)^2 \right]^{1/2} \quad (7)$$

which represents the square root of the sum of squares of the partial derivatives of the data reduction equation with respect to each variable multiplied by the square of its uncertainty value ( $U_x$ ).

Dividing the expression in Equation 7 by the production equation  $VOL(t)$ , results in the final uncertainty analysis expression for the flow rate of in situ material in the pipeline:

$$\frac{U_{VOL(t)}}{VOL(t)} = \left[ \left( \frac{U_{\rho_s}}{\rho_s - \rho_w} \right)^2 + \left( \frac{U_{\rho_i}}{\rho_i - \rho_w} \right)^2 + \left( \frac{U_{\rho_w}}{\rho_i - \rho_w} - \frac{U_{\rho_w}}{\rho_s - \rho_w} \right)^2 + \left( \frac{U_V}{V} \right)^2 + \left( \frac{2U_D}{D} \right)^2 \right]^{1/2} \quad (8)$$

This same procedural method is followed for determining the uncertainty equation for the remaining three production equations. The final uncertainty analysis expression for the solids flow rate is:

$$\frac{U_{M(t)}}{M(t)} = \left[ \left( \frac{U_{\rho_s}}{\rho_s - \rho_w} \right)^2 + \left( \frac{U_{\rho_m}}{\rho_m} - \frac{U_{\rho_m}}{\rho_m - \rho_w} \right)^2 + \left( \frac{U_{\rho_w}}{\rho_i - \rho_w} - \frac{U_{\rho_w}}{\rho_s - \rho_w} \right)^2 + \left( \frac{U_V}{V} \right)^2 + \left( \frac{2U_D}{D} \right)^2 \right]^{1/2} \quad (9)$$

The production equations for the solid and in situ volume hopper loads will involve an additional uncertainty analysis. Note that in these expressions, the average density measured in the hopper, ( $\rho_h$ ), is a function of two measured variables, which are the weight of the material in the hopper as measured by the vessel draft transducers and the volume of the material in the hopper as measured by the water level transducers positioned over the hopper. Because of the dependence of the average density of the material in the hopper on the two measured variables, a separate uncertainty analysis must be performed on the following average density equation:

$$\rho = \frac{W_h}{VOL_h} \quad (10)$$

with  $W_h$  the weight of material in the hopper, and  $VOL_h$  the measured volume in the hopper. The result of this analysis will be used in finding the uncertainty of  $\rho_h$  for input into the final uncertainty analysis expression for the hopper production calculations.

Substituting Equation 10 into Equation 3 results in the following expression for the in situ volume load:

$$VOL = \frac{W_h - \rho_w VOL_h}{\rho_i - \rho_w} \quad (11)$$

with the hopper solids load defined as:

$$M = \frac{W_h - \rho_w VOL_h}{\rho_m - \rho_w} \rho_m \quad (12)$$

Applying the uncertainty analysis procedures to the previous production equations results in the uncertainty expressions for hopper loads. The in situ volume uncertainty relationship is represented by:

$$\begin{aligned} \frac{U_{VOL}}{VOL} = & \left[ \left( \frac{\rho_h}{\rho_h - \rho_w} \right) \left( \frac{U_{W_h}}{W_h} \right)^2 + \left( \frac{-\rho_w}{\rho_h - \rho_w} \right) \left( \frac{U_{V_h}}{V_h} \right)^2 \right. \\ & \left. + \left( \frac{U_{\rho_w}}{\rho_i - \rho_w} - \frac{U_{\rho_w}}{\rho_h - \rho_w} \right)^2 + \left( \frac{U_{\rho_i}}{\rho_i - \rho_w} \right)^2 \right]^{1/2} \end{aligned} \quad (13)$$

The final uncertainty analysis expression for the solids content in the hopper is:

$$\frac{U_M}{M} = \left[ \left( \frac{-\rho_h}{\rho_h - \rho_w} \right) \left( \frac{U_{w_h}}{W_h} \right)^2 + \left( \frac{\rho_w U_{\rho_m}}{\rho_m (\rho_m - \rho_w)} \right)^2 + \left( \frac{U_{p_w}}{\rho_i - \rho_w} - \frac{U_{p_w}}{\rho_h - \rho_w} \right)^2 + \left( \frac{U_{p_i}}{\rho_i - \rho_w} \right)^2 \right]^{1/2} \quad (14)$$

**Evaluating Variable Uncertainties.** The equations are now in the form to insert the variable values and their associated uncertainties for calculating the percent uncertainty in production. In practice, to obtain a reliable estimate of production uncertainty, comprehensive data on the project area sediment and water properties should be collected. If sediment and water samples are taken over the project area, a statistical analysis can be performed to determine the uncertainty in the mean value of the variables used in the uncertainty calculation. Assuming that the variation of the sample values follow a normal (Gaussian) distribution, confidence intervals can be defined for the sample population. Based on a 95 percent confidence interval, the precision limit can be calculated for the sample population. The precision limit (*PL*) is defined as:

$$PL = t \left( \frac{\sigma_{N-1}}{\sqrt{N}} \right) \quad (15)$$

with

$t$  = the  $t$  distribution value

$N$  = number of samples

$\sigma_{N-1}$  = the standard deviation

For example, 10 in situ density measurements are made ( $N=10$ ). The mean value was 1.913 g/cm<sup>3</sup>, and the standard deviation ( $\sigma_{N-1}$ ) was calculated to be 0.068312 g/cm<sup>3</sup>. For  $N-1$  degrees of freedom, and a 95 percent confidence level, the  $t$  distribution value is 2.262 (Coleman and Steel 1989). Therefore the precision limit value is calculated to be 0.04887 g/cm<sup>3</sup>, and the uncertainty for the mean value of the in situ density measurements is 1.913 ± 0.04887 g/cm<sup>3</sup>.

**UNCERTAINTY ANALYSIS EXAMPLE CASES:** To illustrate the utility of the uncertainty analysis method, sample problems will be solved using production instrumentation and vessel specifications from an example pipeline and hopper dredge. In the examples, it is assumed that an adequate number of samples were taken to statistically define the variable uncertainties.

**Example Pipeline Dredge Specifications.** The production meter system on the example pipeline dredge consists of a nuclear density gauge and a magnetic flow meter. The nominal dredge pipe inside diameter ( $D$ ) is 0.61 m and the average flow velocity in the pipe ( $V$ ) is 4.57 m/sec. It is assumed that the pipe diameter uncertainty is ±0.00025 m ( $U_D$ ). For this example, it is assumed



that the dredged material consists of a homogeneous fine sediment mixture, therefore, the uncertainty of a properly calibrated nuclear density gauge as stated by the manufacturer is  $\pm 0.001 \text{ g/cm}^3$  ( $U_{\rho_s}$ ). The magnetic flow meter has a full scale value of 9.76 m/sec and an uncertainty of  $\pm 0.25$  percent of full scale or 0.024 m/sec ( $U_V$ ).

**Example Hopper Dredge Specifications.** The hopper dredge is equipped with differential pressure transducers for measuring the vessel draft, and ultrasonic water level sensors above the hopper for measuring the slurry level in the hopper. The vessel draft measured by the pressure transducers is used to determine the weight of material in the hopper by comparing the measurement to a draft versus vessel weight diagram (Carene diagram). The hopper volume is calculated by comparing the ultrasonic water level measurements of the hopper slurry level to data relating the hopper depth to hopper volume. For this example hopper dredge, the hopper has an average volume of  $6,122 \text{ m}^3$  ( $VOL_h$ ), with an average water load of  $272,160 \text{ kg}$  ( $W_h$ ). The uncertainty of the sensors for measuring hopper depth (volume) and load were calibrated using water in the hopper. Statistical data on the water level transducers for measuring the volume as a function of depth in the hopper indicate a measurement uncertainty of  $\pm 61.22 \text{ m}^3$  ( $U_{VOL_h}$ ), and the differential pressure transducers used for measuring the draft of the vessel have a measurement uncertainty of  $\pm 2,722 \text{ kg}$  ( $U_{W_h}$ ).

**Low Uncertainty Assumptions.** Properly calibrated instruments will have accuracies as previously stated in the dredge specifications. The measured density of the sediment (mineral) and water will be assumed to have uncertainties of  $\pm 0.03 \text{ g/cm}^3$  and  $\pm 0.01 \text{ g/cm}^3$  respectively ( $U_{\rho_m}$  and  $U_{\rho_w}$ ). The in situ density will be assumed to have an uncertainty of  $0.036 \text{ g/cm}^3$  ( $U_{\rho_i}$ ).

**High Uncertainty Assumptions.** For poorly calibrated instruments and unmeasured sediment and water properties, the following uncertainties are applicable. An uncertainty of 5 percent of full scale will be assumed for poorly calibrated hopper monitoring instruments, or  $\pm 13,608 \text{ kg}$  ( $U_{W_h}$ ) and  $\pm 306 \text{ m}^3$  ( $U_{VOL_h}$ ). An uncertainty of  $\pm 0.01 \text{ g/cm}^3$  ( $U_{\rho_s}$ ) is assumed for a poorly calibrated nuclear density probe,  $\pm 0.098 \text{ m/sec}$  ( $U_V$ ) for inaccurate magnetic flow meter calibration,  $\pm 0.03 \text{ g/cm}^3$  ( $U_{\rho_w}$ ) for estimated water density,  $\pm 0.053 \text{ g/cm}^3$  ( $U_{\rho_m}$ ) for inaccurate sediment mineral density, and  $\pm 0.18 \text{ g/cm}^3$  ( $U_{\rho_i}$ ) for estimated sediment in situ density. The variables and their uncertainties for both the pipeline and hopper dredge examples are listed in Tables 1, 2, and 3.

<b>Table 1 Pipeline Dredge Variables and Uncertainties</b>			
<b>Values</b>	<b>Variables</b>		
	<b>D (m)</b>	<b>V (m)</b>	<b><math>\rho_s</math> (g/cm<sup>3</sup>)</b>
Nominal	0.61	4.57	1.20
High Uncertainty (U)	0.00025	0.097	0.01
Low Uncertainty (U)	0.00025	0.0024	0.001

**UNCERTAINTY ANALYSIS EXAMPLE CASES - RESULTS AND DISCUSSION:** To illustrate the error range possible when calculating production with the previously described systems, a series of uncertainty calculations were performed for four example cases. For these

<b>Table 2 Sediment and Water Variables and Uncertainties</b>			
<b>Values</b>	<b>Variables</b>		
	$\rho_m$ (g/cm <sup>3</sup> )	$\rho_i$ (g/cm <sup>3</sup> )	$\rho_w$ (g/cm <sup>3</sup> )
Nominal	2.65	1.80	1.00
High Uncertainty (U)	0.08	0.18	0.03
Low Uncertainty (U)	0.03	0.36	0.01

calculations it is assumed that the hopper dredge is filled to volume (6,122 m<sup>3</sup>) and that the load is at the maximum 272,160 kg. The Case 1 calculation represents the "ideal" situation where the instruments are all calibrated, and the density of the sediments (mineral and in situ) and associated water has been measured (low uncertainty). The Case 2 calculation is for instruments that are

all properly calibrated, but the sediment and water properties are estimated and therefore have a high degree of uncertainty. Case 3 consists of a calculation where the instruments are out of calibration (high uncertainty), and the sediment and water densities are measured. The final case (Case 4) represents the worst case situation, where the instruments are not calibrated, and the sediment and water densities are estimated (high uncertainty). The pipeline dredge uncertainty calculations (volumetric flow rate and solids flow rate) are found in Table 4, and the hopper dredge calculations (in situ volume content and solids content of the hopper) are found in Table 5.

<b>Table 3 Hopper Dredge Variables and Uncertainties</b>			
<b>Values</b>	<b>Variables</b>		
	$\rho_h$ (g/cm <sup>3</sup> )	$V_h$ (m <sup>3</sup> )	$W_h$ (g/cm <sup>3</sup> )
Nominal	1.20	6122.0	272160.0
High Uncertainty (U)	—	306.0	13608.0
Low Uncertainty (U)	—	61.2	2721.6

The pipeline uncertainty calculations reveal two important considerations. First, if the sediment properties are well defined, and the instruments correctly calibrated (the ideal case), an acceptable 6 percent uncertainty in the production of the dredge can be realized. Secondly, the accuracy of the sediment and water properties, primarily the in situ density, are the primary factors in influencing the total accuracy of the calculation. Because of the inherent accuracy of the density and flow velocity instrumentation, the in situ sediment density, with its wide range of uncertainty (conservatively 2 to 10 percent) dominates the accuracy of the calculations. The in situ density can actually vary up to 40 percent (1.2 to 2.0 g/cm<sup>3</sup>) depending on the sediment type and dredging environment. The Case 3 calculation reveals that even when the instruments are out of calibration, the uncertainty (8 percent) is reasonable with known sediment and water properties. The solids flow rate calculations are more accurate because the mineral density of solids only varies to about a maximum of 4 percent. The solids flow rate calculations indicate that the water density is the dominant variable in affecting the production accuracy. The Case 4 calculation reveals an error potential for volumetric production calculations as high as 25 percent if the instruments are not properly calibrated and the sediment and water properties are not well defined.

The hopper production equations reveal the opposite trend. Because the measured density in the hopper is dependent on the measurement of the draft of the vessel (pressure transducers), and the volume in the hopper (ultrasonic water level transducers), the uncertainty in the production calculation is sensitive to the calibration and proper operation of the instruments. For the volume content calculations, the in situ density of the sediments has the greatest influence on the calculation

**Table 4**  
**Summary of Example Pipeline Dredge Uncertainty Calculations**

Percent Uncertainty Contributed by Variable to Total							
Case	$\rho_m$	$\rho_w$	$\rho_i$	$\rho_s$	$V$	$D$	Total Uncertainty
Volumetric Flow Rate							
1	—	2.38	3.43	0.04	0.05	0.001	5.90
2	—	5.03	20.12	0.01	0.01	0.0003	25.17
3	—	1.76	3.13	2.53	0.57	0.0009	7.99
4	—	4.92	19.67	0.97	0.18	0.0003	25.74
Solids Flow Rate							
1	0.10	4.28	—	0.06	0.06	0.002	4.50
2	0.25	13.04	—	0.02	0.02	0.0005	13.33
3	0.07	2.75	—	0.65	0.65	0.001	7.03
4	0.23	12.09	—	0.32	0.32	0.0005	14.37
Case 1 - All instruments calibrated and sediment properties measured. Case 2 - All instruments calibrated and sediment properties estimated. Case 3 - Instruments out of calibration and sediment properties measured. Case 4 - Instruments out of calibration and sediment properties estimated.							

**Table 5**  
**Summary of Example Hopper Dredge Uncertainty Calculations**

Percent Uncertainty Contributed by Variable to Total						
Case	$\rho_m$	$\rho_w$	$\rho_i$	$VOL_h$	$W_h$	Total Uncertainty
Hopper In Situ Volume Content						
1	—	1.44	2.07	2.56	3.68	9.76
2	—	4.80	19.21	0.95	1.37	26.33
3	—	0.36	0.51	15.83	22.79	39.49
4	—	2.72	10.90	13.45	19.37	46.44
Hopper Solids Content						
1	0.05	2.15	—	2.78	4.00	8.98
2	0.22	11.26	—	1.62	2.33	15.43
3	0.01	0.49	—	15.90	22.90	39.30
4	0.08	4.21	—	15.15	21.81	41.25
Case 1 - All instruments calibrated and sediment properties measured. Case 2 - All instruments calibrated and sediment properties estimated. Case 3 - Instruments out of calibration and sediment properties measured. Case 4 - Instruments out of calibration and sediment properties estimated.						

when the instruments are operating properly. However, the error potential becomes very high (40 percent) when the instruments are not operating correctly. For the solids content calculation, the water density is the dominant variable when the instruments are calibrated. These instruments typically operate in an unstable environment, subject to vessel motion and environmental changes (wind, rain, humidity, temperature extremes). Significant error can occur in the calculations (50 percent) if the instruments are not properly calibrated and the sediment properties are not known.

These examples show the utility of the uncertainty analysis procedure in identifying the variables that have the most influence on the production calculation. The variables and their associated uncertainties used in the above examples were chosen to illustrate the procedure. Not all production systems use the same makes or types of instruments, therefore the uncertainties associated with the instrumentation may be different for other applications. Also, the calculations were performed for only one set of dredging conditions (flow velocity, pipe diameter, sediment and water densities). The above examples should only serve as a guide for applying the uncertainty analysis method for determining the accuracy of production system calculations. It should be apparent from the analysis that accurate instrument calibration, along with a thorough knowledge of the properties of the dredged sediments and water is necessary to insure the highest degree of production accuracy.

**CONCLUSIONS:** The following conclusions are based on the uncertainty analysis results:

- Pipeline production meter systems are capable of measuring dredge production to within less than 10 percent, given calibrated instruments and known sediment and water properties. The percent error can be as high as 25 percent if steps are not taken to insure that the instruments are calibrated and material properties known.
- Because of the inherent accuracy of the nuclear density and flow meter instrumentation, the in situ sediment density and the water density have the greatest influence on the accuracy of the in situ volumetric and solids pipeline production calculations.
- For the hopper production calculations, the error potential is greatest for the case of poorly calibrated instruments (40 percent), because the average density measured in the hopper is dependent on two measured variables.
- For hopper production calculations, the in situ sediment density and water density contribute significant error when the instruments are properly calibrated.
- The error in hopper production calculations can range from a low of about 10 percent for calibrated instruments and known sediment and water properties, to almost 50 percent for a worst case of uncalibrated instruments and unknown material properties.

The general uncertainty analysis performed on the production equations reveals the need for determining the correct application and calibration of production monitoring instrumentation, as well as the knowledge of sediment and water properties. The purchase of instrumentation for monitoring production should always be contingent on a thorough training program for dredging personnel. The supplier of the instrumentation should reveal calibration techniques and maintenance schedules necessary for attaining the highest degree of measurement accuracy.

The physical dredging environment should be defined before any dredging occurs. The properties of the sediments and water within the project area must be defined to insure accurate production calculations. Initially, core samples of the sediments should be taken to the maximum dredging depth to identify the sediment type, mineral density, and in situ sediment density. Water properties such as the dissolved and suspended solids content should also be sampled periodically. The data from the coring and water sampling is then used to update the variables in the production equations. For continuous maintenance dredging, these properties should be periodically updated to insure consistency and accuracy in the production data.

**POINTS OF CONTACT:** For additional information on this topic, contact the author, Dr. Steve Scott (601 634-2371, [scotts@wes.army.mil](mailto:scotts@wes.army.mil)), the DOER Instrumentation Focus Area Manager, Mr. James Rosati (601 624-2022, [rosatij@wes.army.mil](mailto:rosatij@wes.army.mil)), or the Program Manager of the Dredging Operations and Environmental Research Program, Dr. Robert M. Engler (601-634-3634, [englerr@wes.army.mil](mailto:englerr@wes.army.mil)). This technical note should be cited as follows:

Scott, S. (2000). "Uncertainty Analysis Applied to Dredge Production Calculation," *DOER Technical Notes Collection* (ERDC TN-DOER-I5), U.S. Army Engineer Research and Development Center, Vicksburg, MS. [www.wes.army.mil/el/dots/doer](http://www.wes.army.mil/el/dots/doer)

## REFERENCES

- Coleman, H. W., and Steele, W. G. (1989). *Experimentation and uncertainty analysis for engineers*. Wiley.
- Peck, R. B., and Hanson, W. E. (1967). *Foundation engineering*. Wiley.
- Rokosch, W. D. (1989). "Measuring dry matter in trailing hopper dredges." *Proceedings from the World Dredging Conference XII*.
- Scott, S. H. (1993). "Uncertainty analysis of dredge production measurement and calculation," *Journal of Waterway, Port, Coastal, and Ocean Engineering* 119(2).

**NOTE:** The contents of this technical note are not to be used for advertising, publication, or promotional purposes. Citation of trade names does not constitute an official endorsement or approval of the use of such products.